CHAPTER 13

Two-Way Independent Analysis of Variance

Summary

A ***factorial ANOVA*** has one dependent variable and two or more independent variables called factors. Each level of one factor is paired with every level of the other factor. A factorial ANOVA tests three null hypotheses: one for each of the two independent variables and one for the interaction between the two independent variables.

Table 13.1 shows a 2 x 3 factorial design. Each participant receives one level of one independent variable and one level of the other independent variable. The participants in the upper left-hand cell of Table 13.1 get Level 1 of Factor A *and* Level 1 of Factor B**.** Their dependent-variable scores are entered into cell A_1B_1.

		Independent Variable *A*			
		Level 1	Level 2	Level 3	Means
Independent Variable *B*	Level 1	Participants' scores (A_1B_1)	Participants' scores (A_2B_1)	Participants' scores (A_3B_1)	$\bar{X}_{B_1}$
	Level 2	Participants' scores (A_1B_2)	Participants' scores (A_2B_2)	Participants' scores (A_3B_2)	$\bar{X}_{B_2}$
	Means	$\bar{X}_{A_1}$	$\bar{X}_{A_2}$	$\bar{X}_{A_3}$	

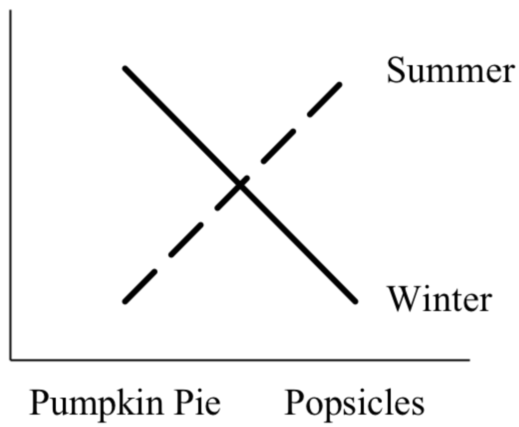
Table 13.1 Illustration of a 2 x 3 Factorial Design

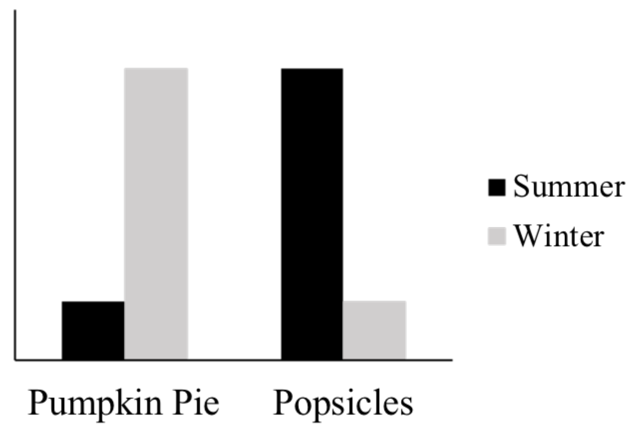
Table 13.1 also shows marginal means for Factor *A* and marginal means for Factor *B***.** The means for Factor *A* are used in *one* of the statistical tests that a factorial ANOVA produces, the main effect of Factor *A*. The marginal means for Factor *B* are involved in a *second* statistical test, the main effect of Factor *B*. Each ***main effect*** is comparable to a one-way independent ANOVA and may be interpreted in the same way as a one-way ANOVA, unless the interaction is significant.

The *third test* in a factorial ANOVA is for an interaction between the two independent variables. ***Interactions*** occur when the effect of one independent variable depends upon which level of the other independent variable you are administering. That is, one independent variable influences the effect that a second independent variable has on the dependent variable.

Perhaps a concrete example of an interaction will help. Which type of dessert is more popular: pumpkin pie or popsicles? It is likely that the answer depends on the time of year. On a hot summer day, a popsicle might be more desirable than a warm slice of pumpkin pie; conversely, on a cold winter day, warm pie might be more appealing. The table and figures that follow present these results of a 2 x 2 factorial design using various formats.

		Dessert Type (*A*)	
		Pumpkin Pie	Popsicles
Time of Year (*B*)	Summer	Low popularity	High popularity
	Winter	High popularity	Low popularity





The effect of dessert type, then, depends on the time of year. In the summer, popsicles are preferred over pumpkin pie; in the winter, the opposite is true. Thus, an interaction is present.

A graph of the cell means is helpful when interpreting the results of a factorial ANOVA. A graph with lines that are not parallel *may* indicate that an interaction has occurred. Likewise, bar graphs with stair steps that change size or direction *may* indicate that two independent variables interact.

In conducting a two-way independent ANOVA test, the total variability of all the scores is partitioned into four components. One component is the variability that is due to the first factor. A second component is the variability due to the second factor. A third component is the variability due to the interaction. The final component is the error variability, which serves as an estimate of the population variability. Each of these components, divided by its degrees of freedom, results in a mean square. The mean square for each main effect and the interaction is then divided by the error mean square to produce *F* values, which are then interpreted using a table of the sampling distribution of *F.* Just like with one-factor repeated measures ANOVA, the appropriate measure of overall effect size is partial eta squared (η_p^2). This time, however, you calculate an overall effect size for *each* effect: main effect of factor *A*, main effect of factor *B*, and the interaction.

As in the preceding chapters, *Tukey HSD* tests may be used to make pairwise comparisons among the levels of a factor. These tests can be conducted for either or for both of the factors in a factorial ANOVA. In Table 13.1, a Tukey *HSD* test on Factor *B* is unnecessary; the *F* test on that factor tells you whether or not the two marginal means are significantly different. Thus, Tukey *HSD* tests are only necessary if a factor has three or more levels. And, because main effects should be interpreted with caution when a significant interaction is present, only conduct Tukey *HSD* tests if the interaction is *not* significant. Similarly, *d* can be used as the estimate of effect size for these pairwise comparisons.

The *assumptions* introduced in Chapter 11 must be met by all data analyzed by ANOVA, including two-way independent ANOVA. These assumptions are that the dependent variable must be *normally distributed* in the populations from which the samples are drawn. The *variances* of the dependent variable must also be *equal* across each population. In addition to these assumptions, the methods described in Chapter 13 require that the *N*'s in each cell be equal, that the scores be independent (not paired or related), and that the levels of the independent variable be chosen by the experimenter rather than at random.

Multiple-Choice Questions

1. As used in analysis of variance, the term factor means ________________.

a. independent variable
b. dependent variable
c. extraneous variable
d. none of the above

2. In terms of the number of factors present in a research design, which two tests are similar to one another?

a. independent-samples *t* test and two-way independent ANOVA
b. paired-samples *t* test and two-way independent ANOVA
c. one-way ANOVA and two-way independent ANOVA
d. none of the above

3. A social psychologist was interested in the effect of propaganda on attitudes of older and younger adults. He measured attitudes toward democracy for all participants, delivered the propaganda, and re-measured attitudes toward democracy. He wanted to analyze the data using a 2 x 2 factorial ANOVA with age as one independent variable and timing of the attitude measure (before vs. after propaganda) as the other. The techniques presented in this chapter will not permit this because ________________.

a. the scores in each cell are not independent
b. an independent variable was not included in the study
c. the dependent variable was normally distributed
d. the levels of the independent variable were not chosen at random by the experimenter

4. A cell in a factorial design refers to ________________.

a. one level of the independent variable
b. one level of the dependent variable
c. one level of one independent variable and one level of a second independent variable
d. all the participants in the experiment

5. When the cell means of a factorial design are presented as a line graph, a nonsignificant interaction is indicated by ________________.

a. parallel lines
b. crossed lines
c. values of *F* greater than 1.00
d. all of the above

6. Suppose you had a 2 x 2 independent factorial ANOVA with the four cell means and you knew that *N* was 6 for each cell. With this information you could calculate ________________.

a. df_A
b. df_{AB}
c. df_{error}
d. all of the above

7. A 3 x 5 factorial ANOVA has ________________ independent variables.

a. 2
b. 3
c. 5
d. 15

8. In a 4 x 4 independent factorial design with five scores per cell, the *df* for the interaction (F_{AB}) would be ________________.

a. 3, 70
b. 6, 64
c. 9, 64
d. none of the above

9. The term *main effect* refers to a comparison of ________________.

a. means for one factor
b. interactions between factors
c. both a. and b.
d. neither a. nor b.

10. An interaction means that ________________.

a. the cell means all differed from one another
b. the margin means for Factor A were identical
c. there was a difference between two variables
d. the effect of one factor depended on the level of the other factor

11. Researchers sometimes subtly expose participants to stimuli (a technique called *priming*) as a way to affect thoughts, feelings, or behaviors. Imagine researchers subtly exposed participants to either happy faces or sad faces (prime type), and then asked them to watch and rate their enjoyment of either a happy film clip or a sad film clip (film type). Which of the following statements would describe an interaction?

a. Film type affected ratings of enjoyment, but prime type did not.
b. Neither film type nor prime type affected ratings of enjoyment.
c. Both film type and prime type affected ratings of enjoyment.
d. Film type affected ratings of enjoyment, but only for those primed with sad faces.

12. “The means of the populations from which the samples were drawn are identical.” This is a statement ________________.

a. of a null hypothesis
b. of one of the assumptions required of data analyzed with a two-way independent ANOVA
c. about an interaction
d. none of the above

13. Which of the following is (are) true?

a. $df_{A} + df_{B} = df_{AB}$
b. $MS_{A} + MS_{B} = MS_{AB}$
c. $SS_{cells} + SS_{error} = SS_{tot}$
d. none of the above

14. Two-way independent ANOVA has which of the following assumptions in common with one-way ANOVA?

a. the dependent variable is assumed to be normally distributed
b. the variances of the dependent-variable scores are equal across populations
c. both a. and b.
d. neither a. nor b.

15. One difference between one-way ANOVA and two-way independent ANOVA is that two-way independent ANOVA ________________.

a. has two or more dependent variables
b. has one independent variable with more than 2 levels
c. requires fewer assumptions about the data
d. has two independent variables

16. Suppose a 5 x 6 independent factorial ANOVA with 7 scores per cell produced $MS_A = 25$, $MS_B = 30$, $MS_{AB} = 15$, and $MS_{error} = 10$ (where Factor A has 5 levels and Factor B has 6 levels). At an alpha level of .01, a Tukey *HSD* ________________.

a. is not appropriate in this instance
b. would be significant for a difference of 2.00 between two Factor A means
c. would be significant for a difference of 2.40 between two Factor A means
d. none of the above

17. Suppose a 2 x 4 independent factorial ANOVA with 5 scores per cell produced $MS_A = 200$, $MS_B = 285$, $MS_{AB} = 295$, and $MS_{error} = 100$ (where Factor A has 2 levels and Factor B has 4 levels). At an alpha level of .05, a Tukey *HSD* ________________.

a. is not appropriate in this instance
b. would be significant for a difference of 10 between two Factor B means
c. would be significant for a difference of 15 between two Factor B means
d. none of the above

18. Which of the following would be reason enough not to calculate pairwise effect size estimates for individual factors?

a. a significant interaction
b. a significant main effect
c. either a. or b.
d. neither a. nor b.

19. A follow-up Tukey *HSD* is not necessary for main effects when the *F* value is based on ________________ degrees of freedom.

a. 1, 20
b. 2, 20
c. 4, 20
d. 4, 10

20. Consider a two-way independent ANOVA in which the dependent variable is reaction time scores. Three different drugs are tested on both older and younger adults. The researchers concluded that the effect of a drug did not depend on whether a person taking it was older or younger. The ANOVA would certainly show that ________________.

a. there is no main effect for drug
b. there is no main effect for age
c. there is no interaction
d. all of the above

Short Answer

1. Describe what a significant main effect tells us and what a significant interaction tells us.

2. In a factorial ANOVA design, when are Tukey *HSD* tests appropriate? When are they not appropriate?

Interpretations

1. The numbers below are cell means. For each set of means, indicate whether the interaction and the main effects are not significant or possibly significant.

a.

	B_1	B_2	B_3
A_1	5	10	15
A_2	5	10	15

b.

	B_1	B_2	B_3
A_1	15	10	15
A_2	5	10	5

c.

	B_1	B_2
A_1	5	10
A_2	10	5

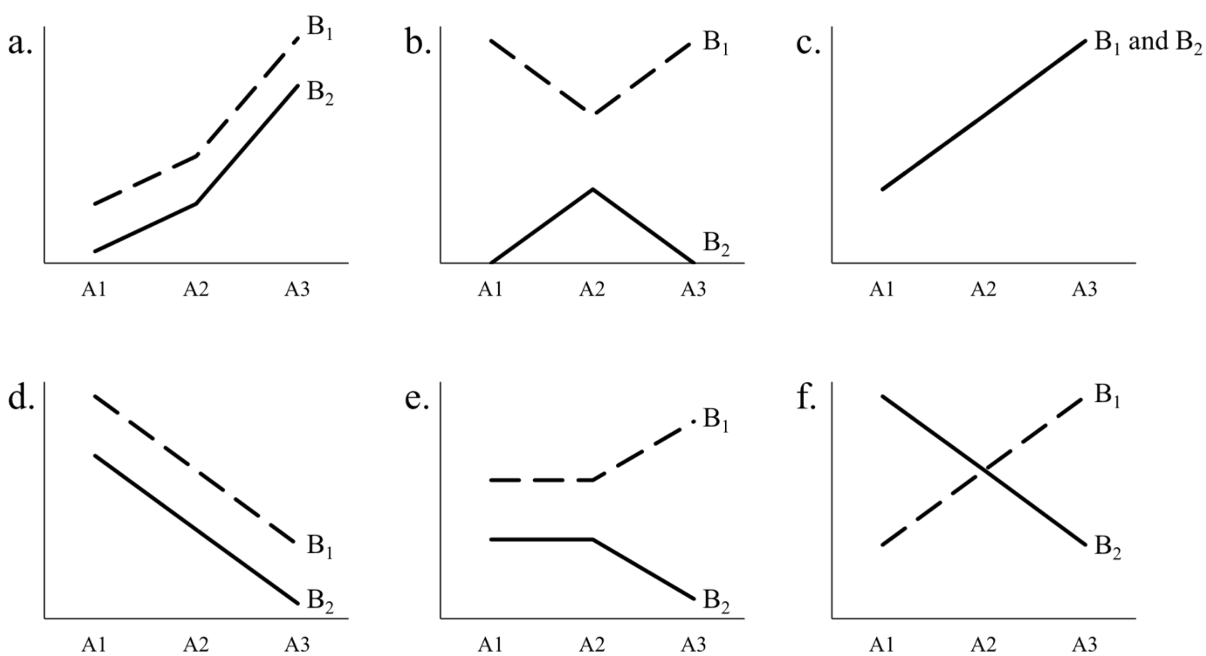
d.

	B_1	B_2
A_1	5	10
A_2	5	10

e.

	B_1	B_2
A_1	5	5
A_2	5	10

2. For each figure, indicate whether the interaction and the main effects are not significant or possibly significant.



3. Across five studies, Makhanova, McNulty, and Maner (2017) researched the psychology of selfies. In Study 2, they examined whether the gender of the person taking the selfie and the gender of the people who would view the selfie would influence the angle at which the photograph was taken. They predicted that women would be more likely to take selfies from a downward angle, which would make them appear smaller, whereas men would be more likely to take selfies from an upward angle, which would make them appear larger. Research assistants approached students on a college campus and asked each of them to take a picture of themselves on a provided camera. Before taking the picture, participants were told that their photo would be shown to either same-gender or other-gender viewers (men for women; women for men). The research assistants coded the angle at which the selfies were taken using a scale ranging from -2 (holding the camera far below the face; low, upward angle) to +2 (holding the camera far above the face; high, downward angle), with 0 representing holding the camera at eye level. Below are modified summary data from Study 2 of Makhanova et al. (2017; retrieved from https://osf.io/edx28/ on July 14, 2018). Set alpha at .05 and find the appropriate critical values. Use the data to write a paragraph summarizing the findings.

		Participant Gender (A): Women	Participant Gender (A): Men	
Viewer Gender (B)	Same gender	-0.16	-0.45	$\bar{X}_{B1} = -0.31$
	Other gender	0.27	0.05	$\bar{X}_{B2} = 0.16$
		$\bar{X}_{A1} = 0.05$	$\bar{X}_{A2} = -0.20$	

Source	*SS*	*df*	*MS*	*F*	η_p^2
Participant Gender (A)	3.95	1	3.95	4.25	.02
Viewer Gender (B)	13.21	1	13.21	14.20	.05
AB Interaction	0.06	1	0.06	0.06	< .001
Error	229.63	246	0.93		
TOTAL	245.85	249			

Problems

1. In a series of studies, O'Brien and Roney (2017) studied people's predictions about and actual experiences of enjoyment of leisure tasks performed before or after completing a work task. In their second study, participants were given descriptions of two tasks: one leisure task (called the *Magic Maker* task) and one tedious task (called the *Fixed Labor* task). Half of the participants actually completed both tasks ("experiencers") and half of the participants only imagined completing both tasks ("predictors"). Additionally, half of the participants played (or imagined playing) the leisure task first and half of the participants played (or imagined playing) the leisure task second. The dependent variable was how much participants rated their experienced or predicted enjoyment of playing *Magic Maker*. Below are modified summary data from Study 2 of O'Brien and Roney (2017; retrieved from https://osf.io/xmnyq/ on July 13, 2018) that will allow you to closely replicate the results. Conduct the four steps of NHST for a factorial ANOVA test (set alpha at .05). Calculate overall effect size indices and only calculate Tukey *HSD* tests and pairwise estimates of effect size, if appropriate. Write a paragraph summarizing your findings.

Note. Because sample sizes in the original data set were unequal across cells, participants' data were dropped at random in three out of four conditions for the purposes of the calculations.

		Factor A: Role — Experiencer (A_1)	Factor A: Role — Predictor (A_2)
Factor B: Task Order	Leisure First (B_1)	$\Sigma X_{A1B1} = 117.67$ $\Sigma X^2_{A1B1} = 441.89$ $N = 40$	$\Sigma X_{A2B1} = 46.00$ $\Sigma X^2_{A2B1} = 246.44$ $N = 40$
	Leisure Second (B_2)	$\Sigma X_{A1B2} = 121.00$ $\Sigma X^2_{A1B2} = 545.89$ $N = 40$	$\Sigma X_{A2B2} = 113.33$ $\Sigma X^2_{A2B2} = 442.00$ $N = 40$

2. Maglio and Polman (2014) studied the effects of spatial orientation on estimates of psychological distance. In their second study, researchers approached people at a subway station in Toronto, Ontario who were either on the eastbound platform (headed *toward* the University of Toronto campus) or the westbound platform (heading *away* from the University of Toronto campus). Researchers then asked each participant to either face toward or away from the University of Toronto and rate how far away they were from the campus on a scale of 1 (*very close*) to 7 (*very far*). Below are summary data from Study 2 of Maglio and Polman (2014; retrieved from https://osf.io/7rajd/ on July 13, 2018) that will allow you to replicate the results. Conduct a factorial ANOVA test—including Tukey *HSD* tests and estimates of effect size, as appropriate—and write a paragraph summarizing the findings. Set alpha at .05.

		Factor A: Direction Traveling	
		Toward Campus (A_1)	Away from Campus (A_2)
Factor B: Direction Facing	Toward Campus (B_1)	$\Sigma X_{A1B1} = 73$ $\Sigma X^2_{A1B1} = 285$ $N = 20$	$\Sigma X_{A2B1} = 86$ $\Sigma X^2_{A2B1} = 396$ $N = 20$
	Away from Campus (B_2)	$\Sigma X_{A1B2} = 72$ $\Sigma X^2_{A1B2} = 280$ $N = 20$	$\Sigma X_{A2B2} = 80$ $\Sigma X^2_{A2B2} = 342$ $N = 20$

ANSWERS

Multiple-Choice Questions

1. a
2. d
   Explanation: A two-way independent ANOVA design involves two independent variables (factors), whereas independent-samples *t* tests, paired-samples *t* tests, and one-way ANOVA designs all involve only one independent variable.
3. a
   Explanation: In this design, one of the factors (timing of the attitude measure) was a repeated measure. The techniques presented in Chapter 13 require that all observations be independent of one another.
4. c
5. a
6. d
7. a
   Explanation: This notation tells you that there are two independent variables. The first independent variable has three levels and the second independent variable has five levels. There are 15 different cells in this design, but only two independent variables.
8. c
   Explanation: The first *df* is for the interaction term and the second *df* is for the error term. N_{tot} is found by multiplying the number of scores per cell (5) by the number of cells (16).
9. a
10. d
11. d
    Explanation: Response option a. describes a main effect of film type, with no main effect of prime type. Response options b. and c. describe no main effects and two main effects, respectively. Option d. describes an effect of film type that *depends on* the level of prime type. An interaction occurs when the effect of one factor depends on the level of another factor.
12. a
13. c
14. c
15. d
16. c
    Explanation: Because the interaction is not significant ($F_{AB} = 1.50$, $p > .01$), Tukey *HSD* tests *are* permissible. With $s_{\bar{X}} = \sqrt{\frac{MS_{error}}{N_t}} = \sqrt{\frac{10}{42}} = 0.49$, a mean difference of 2.00 results in an *HSD* value of 4.08 and a mean difference of 2.40 results in an *HSD* value of 4.90. The critical *HSD* value for $df_{error} = 180$ (use 120 in Table G) and 5 levels of Factor A is 4.71. Thus, a mean difference of 2.00 is not significant, but a mean difference 2.40 is significant. Note that N_t is equal to the total number of scores at one level of A, which would be the sample size of one cell (7) multiplied by the number of rows (6).
17. a
    Explanation: Because the interaction is significant ($F_{AB} = 2.95$, $p < .05$), Tukey *HSD* tests are not appropriate to conduct.
18. a
    Explanation: When a significant interaction is present, interpretation of main effects is typically not recommended.
19. a
    Explanation: The *df* for the numerator is 1 when a factor has only two levels. When a factor has only two levels, the *F* test alone tells you about the difference between the two means.
20. c
    Explanation: When the effect of one independent variable *does not depend* on the level of the second independent variable, an interaction is not present. There is not enough information to determine the possible significance of main effects in this question.

Short Answer

1. A significant main effect indicates mean levels of *one* independent variable significantly differ from the grand mean. In other words, a significance test for a main effect tells us the probability of the means from every level of one factor coming from the same population. If $p < .05$, then it's unlikely all means for one factor came from the same population. With significant interactions, however, it is necessary to talk about the *joint effect* of two or more independent variables. In other words, a significant interaction indicates that

the effect of one factor *depends* on the effect of another factor.

2. Tukey *HSD* tests are appropriate to calculate in factorial designs when you have a significant main effect for a factor that has more than two levels. If a factor has only two levels, a Tukey *HSD* test is not necessary. If a significant interaction is present, Tukey *HSD* tests are not appropriate to calculate.

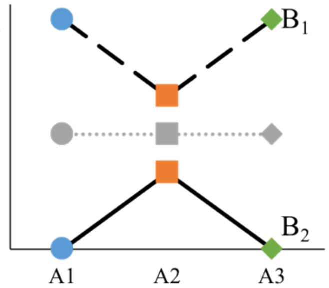
Interpretations

1.
    a. AB interaction: not significant; Main effect of A: not significant; Main effect of B: possibly significant
    b. AB interaction: possibly significant; Main effect of A: possibly significant; Main effect of B: not significant
    c. AB interaction: possibly significant; Main effect of A: not significant; Main effect of B: not significant
    d. AB interaction: not significant; Main effect of A: not significant; Main effect of B: possibly significant
    e. AB interaction: possibly significant; Main effect of A: possibly significant; Main effect of B: possibly significant

    A helpful strategy when answering questions like these is to actually graph the data. Plot one factor on the *x*-axis, the scores on the *y*-axis, and represent the other factor with different lines or patterns. With a line graph, you can make inferences about interactions based on whether the lines are parallel (no significant interaction) or not parallel (possible significant interaction). For inferences about main effects, it's often easiest to look at means. Using part a. of this question as an example, the mean for A_1 is 10 ([5 + 10 + 15] / 3) and the mean for A_2 is 10. These means are identical, so there is no main effect of Factor A. The mean for B_1 is 5, the mean for B_2 is 10, and the mean for B_3 is 15. These means are different from one another, so a main effect of Factor B is possible.

2.
    a. AB interaction: not significant; Main effect of A: possibly significant; Main effect of B: possibly significant
    b. AB interaction: possibly significant; Main effect of A: not significant; Main effect of B: possibly significant
    c. AB interaction: not significant; Main effect of A: possibly significant; Main effect of B: not significant
    d. AB interaction: not significant; Main effect of A: possibly significant; Main effect of B: possibly significant
    e. AB interaction: possibly significant; Main effect of A: not significant; Main effect of B: possibly significant
    f. AB interaction: possibly significant; Main effect of A: not significant; Main effect of B: not significant

    With a line graph, you can make inferences about interactions based on whether the lines are parallel (no significant interaction) or not parallel (possible significant interaction). Inferences about main effects can sometimes be harder to spot. Using part b. of this question as an example, a main effect of Factor B is possibly significant because the entire line for B_1 (and, thus, the overall mean for B_1) is higher than the entire line for B_2. The nonsignificant main effect of A can be harder to spot. In graph that follows, the cell means for A_1 are represented by blue circles, the cell means for A_2 are represented by orange squares, and the cell means for A_3 are represented by green diamonds. If you were to visually approximate the mean of both blue circles, the mean of both orange squares, and the mean of both green diamonds, you would get the gray markers in the middle of the graph, which are all at the same level. Because they are at the same level, there is no main effect of Factor A present.

b.


3.

Source	SS	df	MS	F	η_p^2
Participant Gender (A)	3.95	1	3.95	4.25	.02
Viewer Gender (B)	13.21	1	13.21	14.20	.05
AB Interaction	0.06	1	0.06	0.06	< .001
Error	229.63	246	0.93		
TOTAL	245.85	249			

Critical values: $F_{.05}(1, 200) = 3.89$ (same for all three tests)

A 2 (gender of the participant: women vs. men) x 2 (gender of the viewer: same gender vs. other gender) factorial ANOVA was conducted to test the effects of gender on the angle at which selfies are taken. There was no significant interaction between gender of the participant and gender of the viewer on photograph angles, $F(1, 246) = 0.06$, $p > .05$, $\eta_p^2 < .001$ (very small). However, there were significant main effects of participant gender, $F(1, 246) = 4.25$, $p < .05$, $\eta_p^2 = .02$, and viewer gender, $F(1, 246) = 14.20$, $p < .05$, $\eta_p^2 = .05$. On average, the angle of the photographs were higher (more downward) both when taken by women ($\overline{X} = 0.05$) and when taken for other-gender viewers ($\overline{X} = 0.16$). This is in contrast to the average angle of photographs taken by men ($\overline{X} = -0.20$) and for same-gender viewers ($\overline{X} = -0.31$), which tended to be lower (more upward). The magnitude of these effects were small and medium, respectively. Thus, how people take selfies depends more on the gender of who will view the photo than the gender of who is taking the photo.

For this problem, note that Table F in the textbook does not provide a critical value for $df = 1, 246$. When that happens, always use the next lowest df option, which is $df = 1, 200$ for this problem. Although we used the critical value associated with $df = 1, 200$, report the actual df in the interpretation. Additionally, because each factor has only two levels, Tukey *HSD* tests were not necessary or provided.

Problems

1.

Step 1: Set the null and alternative hypotheses

Role — H_0: $\mu_{\text{experiencer}} = \mu_{\text{predictor}}$
H_1: $\mu_{\text{experiencer}} \neq \mu_{\text{predictor}}$

Task order — H_0: $\mu_{\text{leisure first}} = \mu_{\text{leisure second}}$
H_1: $\mu_{\text{leisure first}} \neq \mu_{\text{leisure second}}$

Interaction — H_0: There will be no interaction.
H_1: There will be an interaction.

Step 2: Set alpha and locate the corresponding critical values

$\alpha = .05$; $F(1, 150) = 3.90$

Step 3: Calculate the appropriate statistics

Source	*SS*	*df*	*MS*	*F*	*p*
Role (A)	39.343	1	39.343	10.40	< .05
Task Order (B)	31.204	1	31.204	8.25	< .05
AB Interaction	25.601	1	25.601	6.77	< .05
Error	590.047	156	3.782		
TOTAL	686.195	159			

		Role (A) Experiencer	Role (A) Predictor	
Task Order (B)	Leisure First	2.942	1.150	$\bar{X}_{B1} = 2.046$
	Leisure Second	3.025	2.833	$\bar{X}_{B2} = 2.929$
		$\bar{X}_{A1} = 2.983$	$\bar{X}_{A2} = 1.992$	$\bar{X}_{GM} = 2.488$

$$SS_{tot} = \sum {X_{tot}}^2 - \frac{(\sum X_{tot})^2}{N_{tot}} = 1{,}676.22 - \frac{(398)^2}{160}$$
$$= 1{,}676.22 - 990.025 = 686.195$$

$$SS_{cells} = \sum\left[\frac{(\sum X_{cell})^2}{N_{cell}}\right] - \frac{(\sum X_{tot})^2}{N_{tot}}$$
$$= \left[\frac{(117.67)^2}{40}\right] + \left[\frac{(46)^2}{40}\right] + \left[\frac{(121)^2}{40}\right] + \left[\frac{(113.33)^2}{40}\right] - \frac{(398)^2}{160}$$
$$= 346.156 + 52.9 + 366.025 + 321.092 - 990.025 = 96.148$$

$$SS_{role} = \sum\left[\frac{(\sum X_A)^2}{N_A}\right] - \frac{(\sum X_{\text{tot}})^2}{N_{tot}}$$
$$= \left[\frac{(238.67)^2}{80}\right] + \left[\frac{(159.33)^2}{80}\right] - \frac{(398)^2}{160}$$
$$= 712.042 + 317.326 - 990.025 = 39.343$$

$$SS_{task\ order} = \sum\left[\frac{(\sum X_B)^2}{N_B}\right] - \frac{(\sum X_{\text{tot}})^2}{N_{tot}}$$
$$= \left[\frac{(163.67)^2}{80}\right] + \left[\frac{(234.33)^2}{80}\right] - \frac{(398)^2}{160}$$
$$= 334.848 + 686.381 - 990.025 = 31.204$$

$$SS_{AB} = N_{cell}\sum[(\bar{X}_{AB} - \bar{X}_A - \bar{X}_B + \bar{X}_{GM})^2]$$
$$= 40\begin{bmatrix} (2.942 - 2.983 - 2.046 + 2.488)^2 + (1.15 - 1.992 - 2.046 + 2.488)^2 \\ +(3.025 - 2.983 - 2.929 + 2.488)^2 + (2.833 - 1.992 - 2.929 + 2.488)^2 \end{bmatrix}$$
$$= 40[(0.401)^2 + (-0.4)^2 + (-0.399)^2 + (0.4)^2] = 40(0.64) = 25.60$$

Check: $SS_{AB} = 96.148 - 39.343 - 31.204 = 25.601$

**Note: Because of rounding, your calculations for SS_{AB} don't perfectly match. Use 25.601 in your ANOVA table.*

$$SS_{error} = \sum\left[\sum X_{cell}{}^2 - \frac{(\sum X_{cell})^2}{N_{cell}}\right]$$
$$= \left[441.89 - \frac{(117.67)^2}{40}\right] + \left[246.44 - \frac{(46)^2}{40}\right] + \left[545.89 - \frac{(121)^2}{40}\right] + \left[442 - \frac{(113.33)^2}{40}\right]$$
$$= [441.89 - 346.156] + [246.44 - 52.9] + [545.89 - 366.025] + [442 - 321.092]$$
$$= [95.734] + [193.54] + [179.865] + [120.908] = 590.047$$

Check: SS_{total} = 96.15 + 590.05 = 686.20

$$df_{tot} = N_{tot} - 1 = 160 - 1 = 159$$
$$df_{role} = A - 1 = 2 - 1 = 1$$
$$df_{task\ order} = B - 1 = 2 - 1 = 1$$
$$df_{AB} = (A-1)(B-1) = (1)(1) = 1$$
$$df_{error} = N_{tot} - (A)(B) = 160 - (2)(2) = 156$$

$$MS_{role} = \frac{SS_{role}}{df_{role}} = \frac{39.343}{1} = 39.343$$

$$MS_{task\ order} = \frac{SS_{task\ order}}{df_{task\ order}} = \frac{31.204}{1} = 31.204$$

$$MS_{AB} = \frac{SS_{AB}}{df_{AB}} = \frac{25.601}{1} = 25.601$$

$$MS_{error} = \frac{SS_{error}}{df_{error}} = \frac{590.047}{156} = 3.782$$

$$F_{role} = \frac{MS_{role}}{MS_{error}} = \frac{39.343}{3.782} = 10.40$$

$$F_{task\ order} = \frac{MS_{task\ order}}{MS_{error}} = \frac{31.204}{3.782} = 8.25$$

$$F_{AB} = \frac{MS_{AB}}{MS_{error}} = \frac{25.601}{3.782} = 6.77$$

<u>Step 4: Make a decision about each null hypothesis</u>

For this problem, note that Table F in the textbook does not provide a critical value for *df* = 1, 156. When that happens, always use the next lowest *df* option, which is *df* = 1, 150 for this problem. Although we used the critical value associated with *df* = 1, 150, report the actual *df* in the conclusion.
10.40 > 3.90; reject null hypothesis for Factor A
8.25 > 3.90; reject null hypothesis for Factor B
6.77 > 3.90; reject null hypothesis for interaction

<u>Calculate effect size index for *F*</u>

Role:
$$\eta_p^2 = \frac{SS_{treat}}{SS_{treat} + SS_{error}} = \frac{39.343}{39.343 + 590.047} = 0.06$$

Task order:
$$\eta_p^2 = \frac{SS_{treat}}{SS_{treat} + SS_{error}} = \frac{31.204}{31.204 + 590.047} = 0.05$$

Interaction:

$$\eta_p^2 = \frac{SS_{treat}}{SS_{treat} + SS_{error}} = \frac{25.601}{25.601 + 590.047} = 0.04$$

Because the interaction was significant and each factor has only two levels, Tukey *HSD* tests are not appropriate. And, because the significant interaction was driven largely by one cell (predictors with the leisure task first), further interpretation of the main effects is not recommended.

Write your interpretation:
A 2 (role: experiencer vs. predictor) x 2 (task order: leisure first vs. leisure second) factorial ANOVA was conducted to examine perceived and experienced enjoyment of a leisure activity that came either before or after a work task. There was a small-to-moderate significant interaction between the two factors, $F(1, 156) = 6.77$, $p < .05$, $\eta_p^2 = 0.04$. For those who only imagined completing the tasks, average ratings of expected enjoyment were lower when the leisure task came before a work task than when it followed a work task. For those who actually completed both tasks, though, average ratings of experienced enjoyment were very similar between the two task orders. Thus, although people may expect to enjoy leisure tasks less when a work task remains undone, in actual experience, leisure tasks are similarly enjoyable both before and after work. There were moderate significant main effects for both role, $F(1, 156) = 10.41$, $p < .05$, $\eta_p^2 = 0.06$, and task order, $F(1, 156) = 8.25$, $p < .05$, $\eta_p^2 = 0.05$, as well.

2.

Source	*SS*	*df*	*MS*	*F*	*p*
Direction Traveling (A)	5.512	1	5.512	4.78	< .05
Direction Facing (B)	0.612	1	0.612	0.53	> .05
AB Interaction	0.313	1	0.313	0.27	> .05
Error	87.55	76	1.152		
TOTAL	93.987	79			

A 2 (direction traveling: away vs. toward) x 2 (direction facing: away vs. toward) factorial ANOVA was conducted to test the effects of spatial (direction traveling) and visual (direction facing) orientation on estimates of subjective distance. There was no significant interaction between direction traveling and direction facing on estimates of subjective difference, $F(1, 76) = 0.27$, $p > .05$, $\eta_p^2 = 0.004$, nor was there a significant main effect of direction facing, $F(1, 76) = 0.53$, $p > .05$, $\eta_p^2 = 0.01$. Both effect sizes were small. However, there was a significant main effect of direction traveling, $F(1, 76) = 4.78$, $p < .05$, $\eta_p^2 = 0.06$, such that those traveling toward the campus estimated the campus to be closer ($\bar{X} = 3.63$) than those traveling away from the campus ($\bar{X} = 4.15$). The magnitude of this main effect was medium.

		Direction Traveling (A): Toward Campus	Direction Traveling (A): Away from Campus	
Direction Facing (B)	Toward Campus	3.65	4.30	$\bar{X}_{B1} = 3.975$
	Away from Campus	3.60	4.00	$\bar{X}_{B2} = 3.80$
		$\bar{X}_{A1} = 3.625$	$\bar{X}_{A2} = 4.15$	$\bar{X}_{GM} = 3.888$

$$SS_{tot} = \sum {X_{tot}}^2 - \frac{(\sum X_{tot})^2}{N_{tot}} = 1{,}303 - \frac{(311)^2}{80}$$
$$= 1{,}303 - 1{,}209.013 = 93.987$$

$$SS_{cells} = \sum \left[\frac{(\sum X_{cell})^2}{N_{cell}}\right] - \frac{(\sum X_{tot})^2}{N_{tot}}$$
$$= \left[\frac{(73)^2}{20}\right] + \left[\frac{(86)^2}{20}\right] + \left[\frac{(72)^2}{20}\right] + \left[\frac{(80)^2}{20}\right] - \frac{(311)^2}{80}$$
$$= 266.45 + 369.8 + 259.2 + 320 - 1{,}209.013 = 6.437$$

$$SS_{traveling} = \sum \left[\frac{(\sum X_A)^2}{N_A}\right] - \frac{(\sum X_{\text{tot}})^2}{N_{tot}}$$
$$= \left[\frac{(145)^2}{40}\right] + \left[\frac{(166)^2}{40}\right] - \frac{(311)^2}{80}$$
$$= 525.625 + 688.9 - 1{,}209.013 = 5.512$$

$$SS_{facing} = \sum \left[\frac{(\sum X_B)^2}{N_B}\right] - \frac{(\sum X_{\text{tot}})^2}{N_{tot}}$$
$$= \left[\frac{(159)^2}{40}\right] + \left[\frac{(152)^2}{40}\right] - \frac{(311)^2}{80}$$
$$= 632.025 + 577.6 - 1{,}209.013 = 0.612$$

$$SS_{AB} = N_{cell} \sum [(\bar{X}_{AB} - \bar{X}_A - \bar{X}_B + \bar{X}_{GM})^2]$$
$$= 20 \begin{bmatrix} (3.65 - 3.63 - 3.98 + 3.89)^2 + (4.30 - 4.15 - 3.98 + 3.89)^2 \\ +(3.60 - 3.63 - 3.80 + 3.89)^2 + (4.00 - 4.15 - 3.80 + 3.89)^2 \end{bmatrix}$$
$$= 20[(-0.07)^2 + (0.06)^2 + (0.06)^2 + (-0.06)^2] = 20(0.0157) = 0.31$$

Check: $SS_{AB} = 6.437 - 5.512 - 0.612 = 0.313$

$$SS_{error} = \sum \left[\sum X_{cell}{}^2 - \frac{(\sum X_{cell})^2}{N_{cell}}\right]$$
$$= \left[285 - \frac{(73)^2}{20}\right] + \left[396 - \frac{(86)^2}{20}\right] + + \left[280 - \frac{(72)^2}{20}\right] + \left[342 - \frac{(80)^2}{20}\right]$$
$$= [18.55] + [26.2] + [20.8] + [22] = 87.55$$

Check: $SS_{\text{total}} = 6.437 + 87.55 = 93.987$

$$df_{tot} = N_{tot} - 1 = 80 - 1 = 79$$
$$df_{traveling} = A - 1 = 2 - 1 = 1$$
$$df_{facing} = B - 1 = 2 - 1 = 1$$
$$df_{AB} = (A - 1)(B - 1) = (1)(1) = 1$$
$$df_{error} = N_{tot} - (A)(B) = 80 - (2)(2) = 76$$

$$MS_{traveling} = \frac{SS_{traveling}}{df_{traveling}} = \frac{5.512}{1} = 5.512$$

$$MS_{facing} = \frac{SS_{facing}}{df_{facing}} = \frac{0.612}{1} = 0.612$$

$$MS_{AB} = \frac{SS_{AB}}{df_{AB}} = \frac{0.313}{1} = 0.313$$

$$MS_{error} = \frac{SS_{error}}{df_{error}} = \frac{87.55}{76} = 1.152$$

$$F_{traveling} = \frac{MS_{traveling}}{MS_{error}} = \frac{5.512}{1.152} = 4.78$$

$$F_{facing} = \frac{MS_{facing}}{MS_{error}} = \frac{0.612}{1.152} = 0.53$$

$$F_{AB} = \frac{MS_{AB}}{MS_{error}} = \frac{0.313}{1.152} = 0.27$$

Traveling:

$$\eta_p^2 = \frac{SS_{treat}}{SS_{treat} + SS_{error}} = \frac{5.512}{5.512 + 87.55} = 0.06$$

Facing:

$$\eta_p^2 = \frac{SS_{treat}}{SS_{treat} + SS_{error}} = \frac{0.612}{0.612 + 87.55} = 0.01$$

Interaction:

$$\eta_p^2 = \frac{SS_{treat}}{SS_{treat} + SS_{error}} = \frac{0.313}{0.313 + 87.55} = 0.004$$

Critical values: $F_{.05}(1, 70) = 3.98$ (same for all three tests)

For this problem, note that Table F in the textbook does not provide a critical value for *df* = 1, 76. When that happens, always use the next lowest *df* option, which is *df* = 1, 70 for this problem. Although we used the critical value associated with *df* = 1, 70, report the actual *df* in the conclusion. Additionally, because each factor has only two levels, Tukey *HSD* tests are not appropriate.

References

Makhanova, A., McNulty, J. K., & Maner, J. K. (2017). Relative physical position as an impression-management strategy: Sex differences in its use and implications. *Psychological Science, 28*(5), 567-577. doi: 10.1177/09567716688885

O'Brien, E. & Roney, E. (2017). Worth the wait? Leisure can be just as enjoyable with work left undone. *Psychological Science, 28*(7), 1000-1015. doi: 10.1177/0956797617701749

Romano, A. & Balliet, D. (2017). Reciprocity outperforms conformity to promote cooperation. *Psychological Science, 28*(10), 1490-1502. doi: 10.1177/0956797617714828